Accretion onto a Black Hole in the Presence of Bremsstrahlung Cooling - Parameter Space Study

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ABSTRACT

The dynamics of accretion onto a schwarzschild black hole is studied using Paczynski-Wiita pseudo newtonian potential. Steady state solution of the flow equations is obtained using thin disc approximation, including the effect of bremsstrahlung cooling in the energy equation. The topology of transonic solutions are got and the conditions for shock formation (Rankine-Hugoniot conditions) are checked. Shock and no-shock regions in the parameter space of accretion rate and specific angular momentum are identified. We motivate the time-dependent numerical simulation of the flow, as a candidate for explaining the quasi periodic oscillations observed in black hole candidates.

Subject headings: accretion, accretion disks — black hole physics — hydrodynamics — methods: numerical — shock waves

1. Introduction

The parameter space of shock formation in adiabatic transonic accretion discs was first discussed in Chakrabarti(1989). Though time-dependent numerical simulation of transonic accretion disc in the presence of cooling was dealt in Molteni, Sponholz & Chakrabarti (1996, hereafter MSC96), the detailed study of parameter space was reserved for the future. In this paper we describe the procedure for obtaining the flow topology in the presence of bremsstrahlung cooling and obtain the dependence of topology and shock formation on the parameters, accretion rate and specific angular momentum.

2. Flow equations

The equations which govern the flow are the basic conservation equations of mass, momentum and energy. The pseudo newtonian potential of Paczynski-Wiita (1980) is used, which mimics the gravitational field around a schwarzschild black hole to a sufficient accuracy. The effect of bremsstrahlung cooling (Lang 1980) in electron-proton plasma is included in the energy equation. $\mathbf{v}(v_r, v_\phi, v_z)$, ρ and p are the velocity, density and pressure respectively.

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0 \tag{1}$$

Euler equation:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\frac{\nabla \mathbf{p}}{\rho} - \nabla \mathbf{g} \tag{2}$$

where $g = -\frac{GM}{(r-r_g)}$ is the Paczynski-Wiita potential, G is the gravitational constant, M is the mass of the black hole, $r_g = \frac{2GM}{c^2}$ is the schwarzschild radius and c is the velocity of light.

Energy equation:

$$\nabla.(\rho\epsilon\mathbf{v}) + \mathbf{\Lambda} - \mathbf{\Gamma} = -\frac{\partial}{\partial \mathbf{t}}(\rho\epsilon) \tag{3}$$

where $\epsilon = \frac{1}{2}{v_r}^2 + U + \frac{p}{\rho} + g + \frac{1}{2}{v_\phi}^2$ is the specific energy, $U = \frac{p}{\rho(\gamma-1)}$ is the thermal energy, γ is the adiabatic index, $\Lambda = 1.43 \times 10^{-27} \frac{\rho^2}{m_p^2} T^{1/2} g_f$ is the expression for bremsstrahlung cooling, T is the temperature, m_p is the mass of proton, g_f is the gaunt factor and Γ is the heating term.

Thin disc approximation:

We use cylindrical polar coordinates (r, ϕ, z) for the axisymmetric flow. Because of axisymmetry and thinness of the disc the following assumptions are made.

$\rho(\phi^0, z^0)$ i.e. ρ is not a function of ϕ and z

$v_r(\phi^0, z^0)$

$v_\phi(\phi^0, z^0)$

$v_z(r^0, \phi^0, z^0)$

$p(r)$ i.e. p is a function of r only

$g(r)$

$\epsilon(\phi^0)$

By the help of these assumptions it is possible to write the steady state equations in the following form.

$$\frac{\psi' - a^2/r + jg_f \frac{2}{3} \frac{\rho}{v m_p{}^2} T^{1/2}}{a^2/v - v} = \frac{dv}{dr} \; ; \; \psi = -\frac{GM}{(r - r_g)} + \frac{1}{2} v_\phi{}^2 \tag{4}$$

$$\frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{v}\frac{dv}{dr} + \frac{1}{r} = 0 \tag{5}$$

$$v\frac{dv}{dr} + \frac{2a}{\gamma}\frac{da}{dr} + \frac{a^2}{\rho\gamma}\frac{d\rho}{dr} + \psi' = 0 \tag{6}$$

where $'$ denotes the derivative with respect to r, $j = 1.43 \times 10^{-27}$ in cgs units and the polytropic relation $p = K\rho^\gamma$ is used to obtain the sound speed $a = \sqrt{\frac{\partial p}{\partial \rho}} = \sqrt{\frac{\gamma p}{\rho}}$. These equations are solved using fourth-order Runge-Kutta method (Press et al. 1992).

3. Sonic point analysis

The flow can pass through a point where denominator in the expression for $\frac{dv}{dr}$ becomes zero. If it happens that the numerator also becomes zero, then $\frac{dv}{dr}$ can be finite. We call such a point a critical point. In this problem the critical point is same as sonic point. At critical point, using l' Hospital's rule, we get (for $\gamma = \frac{5}{3}$),

$$\frac{dv}{dr} = \frac{-B \pm \sqrt{(B^2 - 4AC)}}{2A} \tag{7}$$

$$A = \frac{8}{3} \tag{8}$$

$$B = -\frac{1}{3}\frac{v}{r} - \frac{4}{3}\zeta\rho a^{2\alpha-2}(1 + \frac{\alpha}{3}) + \frac{5}{3}\frac{\psi'}{v} \tag{9}$$

$$C = \frac{5}{3}\frac{\psi'}{r} + \psi'' - \frac{2}{3}\zeta\frac{a^{2\alpha-1}}{r}\rho(1-\alpha) - \frac{10}{9}\alpha\rho\zeta a^{2\alpha-3}\psi' \tag{10}$$

$$\zeta = jg_f\frac{1}{m_p^2}(\frac{\mu m_p}{\gamma k})^\alpha \tag{11}$$

where $\alpha = 0.5$, $\mu = 0.5$ and k is the boltzmann constant. Now starting from the critical point we obtain the transonic solution topologies.

4. Numerical scheme

Fluid dynamical problems are inherently sensitive to the boundary conditions. We formulate the problem as follows. The variables which are functions of space and time are $\mathbf{v}, \rho, p$ or $\mathbf{v}, \rho, a$. The parameters are accretion rate, specific angular momentum(λ), γ, cooling process and viscosity. We consider inviscous flow, set $\gamma = 5/3$ and the mass of the black hole M is chosen as $10^8 M_\odot$. So the free parameters are accretion rate and λ. The location of the critical points, r_{c1}, r_{c2}, is the freedom we have, subjected to the constraint that shock conditions should be satisfied. The natural constraint of stability is likely to decide the uniqueness of the solution. The algorithm for obtaining the numerical solution is,

1. Choose accretion rate and λ
2. Choose r_{c1} and r_{c2} range and find corresponding a_{c1} and a_{c2}
3. Obtain solution topology by using fourth-order Runge-Kutta method
4. Check the shock conditions

The typical components of the topology are shown in fig.1, where mach number is plotted as a function of the radial distance. Fig.2 and 3 shows the solution topologies for chosen parameter values. Rankine-Hugoniot shock conditions (Landau & Lifshitz 1984) basically ensure that mass, momentum and energy are conserved in spite of a discontinuity. For infinitesimally thin and non-dissipative shock, the shock conditions take the form,

$$p_1 + \rho_1 {v_1}^2 = p_2 + \rho_2 {v_1}^2, \qquad \frac{1}{2}{v_1}^2 + \frac{{a_1}^2}{\gamma - 1} = \frac{1}{2}{v_2}^2 + \frac{{a_2}^2}{\gamma - 1} \tag{12}$$

where the suffixes 1 and 2 refer to pre-shock and post-shock quantities.

5. Parameter space

We use the following procedure to obtain the parameter space. The accretion rate in eddington units, is varied in the range (.0001, 500) and λ in the units of $\frac{2GM}{c}$, is varied in the range $(1.6, 2.5)$. It is suggested that (Chakrabarti 1996) flows with λ in the range between that of marginally stable and marginally bound orbit would form steady accretion discs, when self-gravity of the disc is neglected. For a chosen accretion rate and λ we scan the r-axis from $1.5r_g$ to $1000r_g$ to find if it can be a critical point. The critical point can be of X or Alp or plA or lA or x type as shown in fig.1. X looks like the English alphabet X, Alp like Greek alphabet alpha which opens towards infinity, plA is reflected Alp which opens towards inner boundary, lA is plA which doesn't reach the inner boundary when λ is high and x is X which doesn't reach the inner boundary when accretion rate is high. For certain parameter values there is an intermediate range of r which cannot be a critical point (denoted by - in topology column of Table 1). Table 1 shows the parameter space. We obtain supersonic branches for outer critical points and subsonic branches for inner critical points. When shock conditions are satisfied the flow makes a transition from supersonic to subsonic branch and reaches the inner boundary (chosen as $1.5r_g$) supersonically. When shock conditions are not satisfied accretion is still possible if X type critical point exists.

6. Discussion and conclusions

Of all the different possible branches of accretion for chosen parameter value, the real flow is likely to choose the branch which is most stable, as perturbations are always present in a real situation. If the assumption of steady flow is relaxed the flow might still choose a steady solution branch if the time scale of change is large. Time-dependent numerical simulation of the flow (MSC96) shows that flow solution oscillates about the steady state solution when certain resonance condition is met. The 'perturbations' which occur in a numerical code and physical perturbations should be related, to increase the faith in numerical results. Such numerical studies will be pursued in the future. The oscillation of shock location (MSC96) would mean the size of post-shock region, which is the source for hard photons (Chakrabarti & Titarchuk 1995), is also oscillating. These would result in quasi periodic oscillations as reported in Chakrabarti & Manickam (2000).

We thank Indian Space Research Organization for funding the project, Quasi-Periodic Oscillations in Black Hole Candidates, of which this work forms a part.

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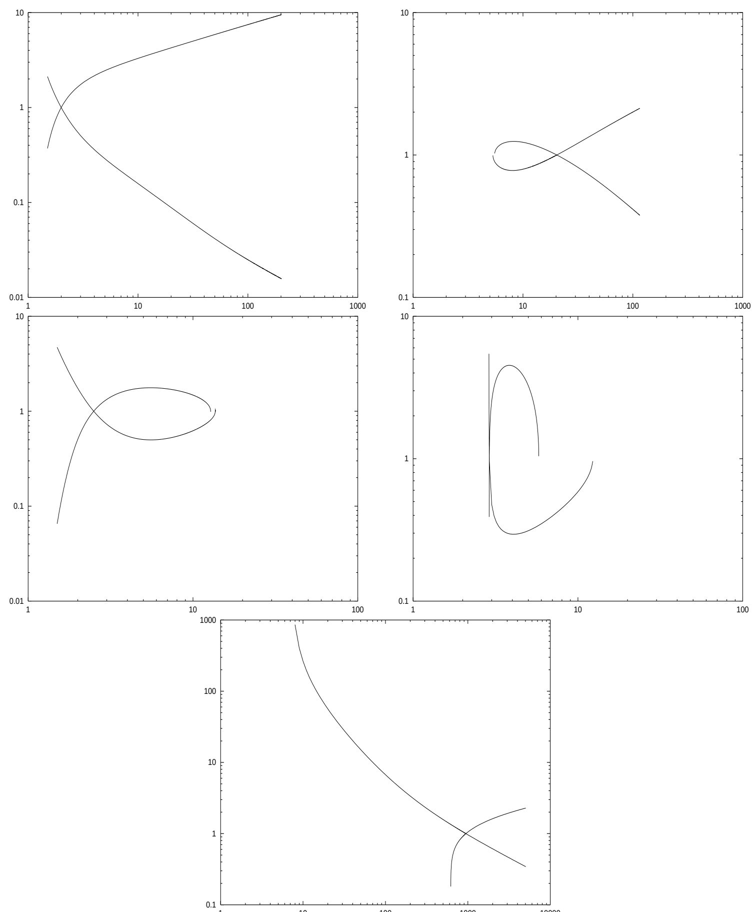
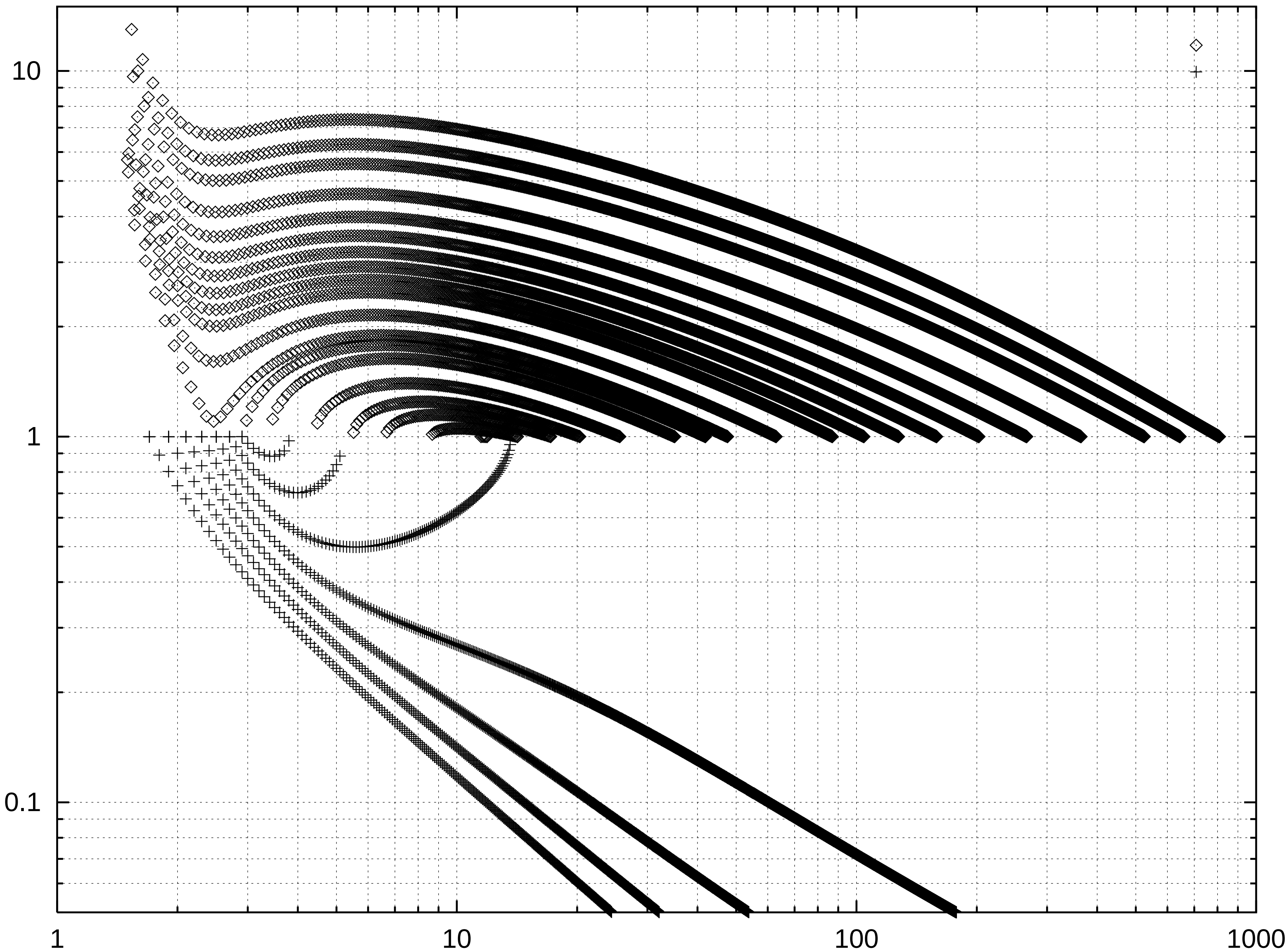


Fig. 1.— Mach number Vs radial distance of X, Alp, plA, lA and x



10
1
0.1
1
10
100
1000

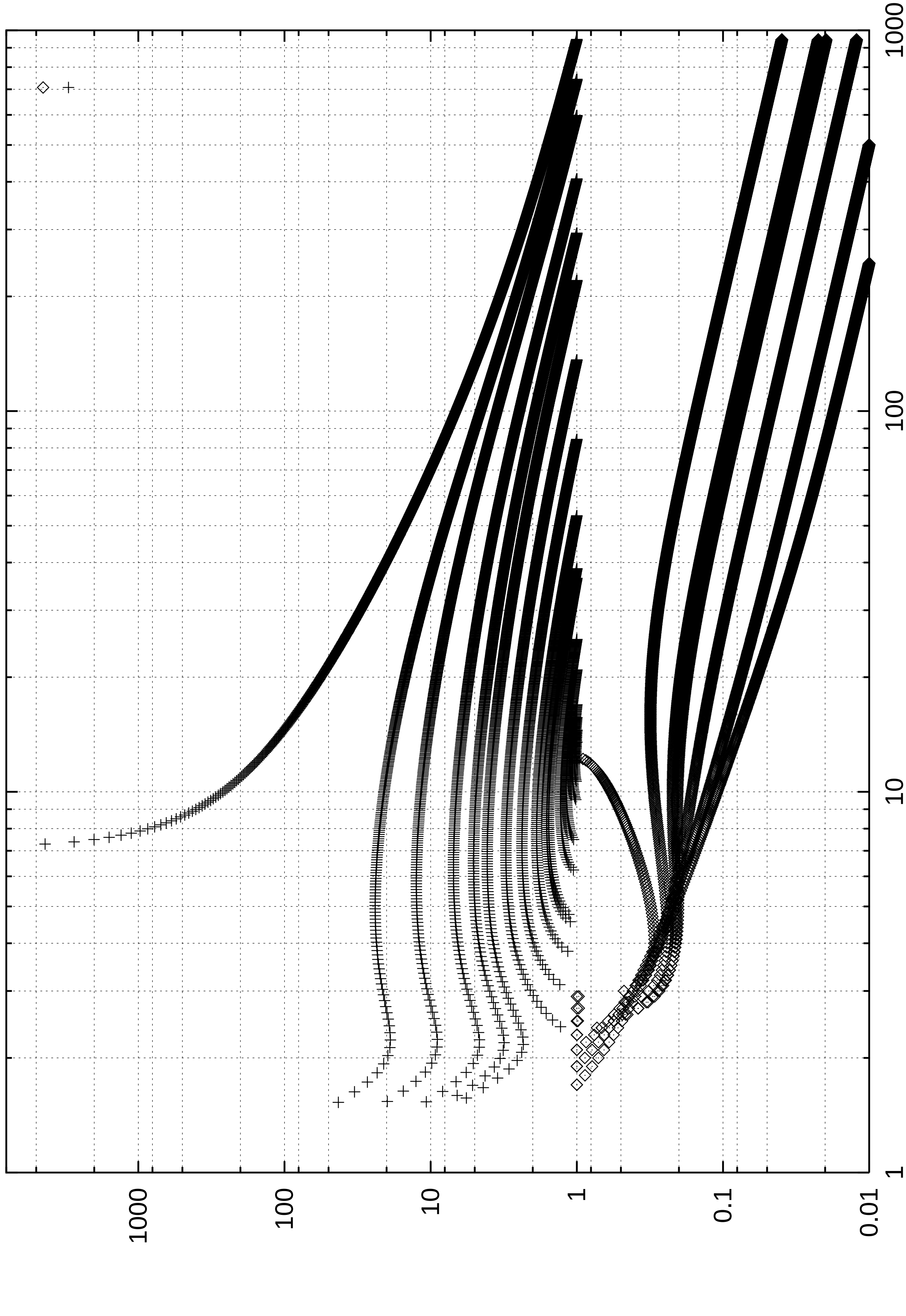

1000
100
10
1
0.1
0.01
1
10
100
1000

Table 1.

accretion rate	λ	topology	shock
.0001	1.6	X	
.0001	1.65	X plA - Alp X	noshock
.0001	1.7	X plA - Alp X	shk
.0001	1.9	X plA - Alp X	shk2
.0001	2.0	X - Alp	noshock
.001	1.6	X	
.001	1.65	X plA - Alp X	shk
.001	1.9	X plA - Alp X	shk2
.001	2.0	X - Alp	noshock
.01	1.6	X	
.01	1.65	X plA - Alp X	shk
.01	1.9	X plA - Alp X	shk2
.01	2.0	X lA - Alp	noshock
.1	1.6	X	
.1	1.65	X plA - Alp X	shk
.1	1.9	X plA - Alp X	shk
.1	2.0	X - Alp	noshock
1.0	1.6	X	
1.0	1.65	X plA - Alp X	shk
1.0	1.7	X plA - Alp X	
1.0	1.8	X plA - Alp X	
1.0	1.9	X - Alp X	shk2
1.0	2.0	X - Alp	noshock
1.0	2.1	X - Alp	
1.0	2.3	X lA - Alp	
1.0	2.4	lA - Alp	
1.0	2.5	lA - Alp	
2.0	1.6	X	
2.0	1.65	X plA - Alp X	shk
2.0	1.8	X plA - Alp X	
2.0	1.9	X lA - Alp X	shk1

Table 1—Continued

accretion rate	λ	topology	shock
2.0	2.0	X lA - Alp	noshock
2.0	2.3	X lA - Alp	
2.0	2.4	x lA - Alp	
5.0	1.6	X	
5.0	1.65	X plA - Alp X x	shk
5.0	1.7	X plA - Alp X	
5.0	1.8	X plA - Alp X	
5.0	1.9	X lA - Alp X x	shk
5.0	2.0	X lA - Alp X x	noshock
5.0	2.3	X x lA - Alp x	
20.	1.65	X plA - Alp X x	
20.	2.4	X x lA - Alp x	
30.	1.65	X plA - Alp X x	
50.	1.65	X x	
50.	1.7	X plA - Alp X x	shk
50.	1.8	X plA - Alp X x	shk1
50.	1.9	X plA lA - Alp X x	shk
50.	2.0	X x lA - Alp X x	noshock
50.	2.4	X x lA - Alp x	
500.	1.6	X x	
500.	1.8	X x	
500.	2.4	x	